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Double Coulomb Excitation with Oxygen Ions*

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Total cross sections for Coulomb excitation of the first and second excited states of the ground-state rotational bands of eight even-even rare earth nuclei have been measured. Oxygen ions accelerated to energies in the range 14 to 50 MeV in the Wisconsin tandem accelerator produced the nuclear excitations. The transitions from the second excited state (4^+) to the first (2^+), and from the first to the ground state (0^+), were detected by analyzing the emitted internal conversion electrons in a wedge-shaped magnetic beta-ray spectrometer. Cross sections for excitation of the 2^+ levels yield $B(E2)$ values in satisfactory agreement with those obtained from lifetime measurements and from inelastic scattering of protons, deuterons, and alpha particles. Cross sections for double $E2$ excitation of the 4^+ levels are in qualitative agreement with the multiple Coulomb excitation theory of Alder and Winther. The energies of the 4^+ levels are Sm^{152}, 366.2 keV; Sm^{154}, 267.1 keV; Gd^{160}, 248.6 keV; Dy^{162}, 266.0 keV; Er^{170}, 261.5 keV; Yb^{176}, 271.9 keV; Hf^{180}, 309.4 keV; and W^{184}, 363.4 keV. In all cases the uncertainty is ±2 keV.

I. INTRODUCTION

FOR Coulomb excitation the use of heavy ions ($Z>2$) as projectiles is advantageous because the Coulomb interaction with the target nucleus can become large while the probability of nuclear penetration remains small. Moreover, the phenomenon of multiple Coulomb excitation, first observed by Newton and Stephens[1] with oxygen ions, can yield information about high spin levels which may be inaccessible by direct excitation. Because the interaction probabilities in multiple Coulomb excitation proved too large to be treated adequately by perturbation theory,[2] Alder and Winther developed a theory of multiple Coulomb excitation[3] using a sudden approximation.

The purpose of the experiment reported here[4] was to test the predictions of Alder and Winther concerning double $E2$ excitation of the spin 4, even parity rotational levels in even-even nonspherical nuclei. Deformed nuclei were chosen because they are expected to exhibit a well-developed ground-state rotational band in which the rotational levels are coupled by large quadrupole transition probabilities.[5] Measurement of the excitations by detection of the internal conversion electrons emitted in the nuclear decays was advantageous because of the high resolution obtainable with a magnetic spectrometer. The experimental techniques used in accelerating oxygen ions, preparing targets, and detecting conversion electrons are discussed below in Sec. II. Experimental results are presented in Sec. III and compared with theory and other experimental data in the final section.

II. EXPERIMENTAL PROCEDURE

Oxygen ions were produced in the duo-plasmatron ion source of the tandem accelerator from a gas mixture consisting of 10% oxygen and 90% hydrogen. The gas was ionized by electrons thermionically emitted from a U-shaped filament of tungsten one millimeter in diameter. Typical filament life in the gas mixture was forty hours. The positive ions were extracted from the source and then converted to negative ions by hydrogen gas in an electron adding canal. Typically 2 to 4 μA of negative ions were obtained for injection into the accelerator. After acceleration into the tandem the negative ions were stripped to various positive ionic charge states by collisions with oxygen gas in the field-free region inside the high potential terminal. After acceleration out of the tandem, oxygen beams of a particular momentum per ionic charge were selected by adjusting the current in a 90° analyzing magnet. Adjustment of the terminal potential, indicated by a generating voltmeter, permitted selection of the desired beam. Target currents and energies obtained in the various charge states are listed in Table I.

Thin targets for bombardment by oxygen ions were prepared by vacuum evaporation of enriched rare-earth isotopes purchased in oxide form from Oak Ridge National Laboratory. Targets were prepared in thicknesses ranging from 5 μg/cm^2 to 1 mg/cm^2. The thinnest targets were ideal for studying the intense, low-energy decay transition from the first excited state. Somewhat thicker targets (200 μg/cm^2) were necessary to obtain reasonable counting rates for the low-intensity transitions from the second to the first excited states. The thickest targets were used to check energy loss corrections discussed below. Thick backings of carbon, aluminum, titanium, copper, and nickel were used. Backings with atomic masses smaller than the lightest

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[3] K. Alder and A. Winther, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, 16 (1953).

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[5] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, No. 16 (1953).

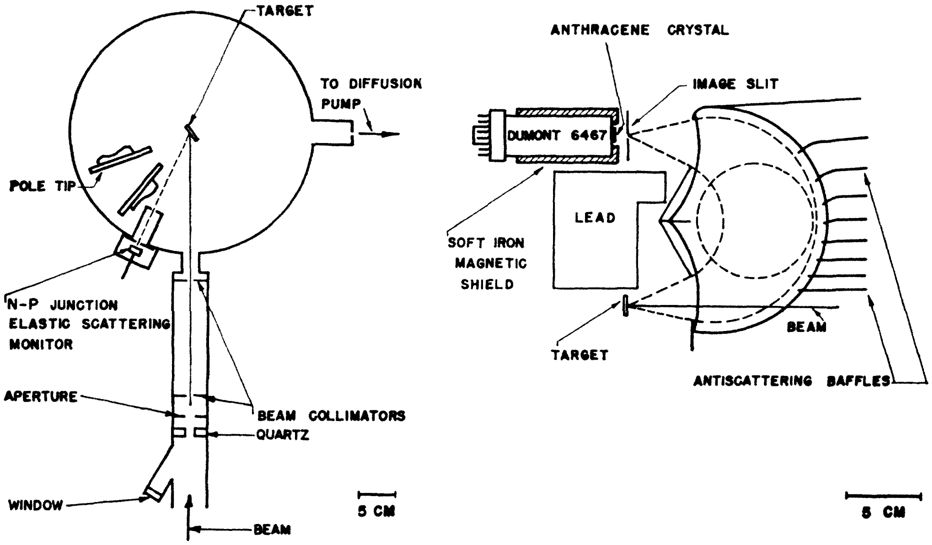

FIG. 1. Schematic of the experimental equipment. Part (A) is in a horizontal plane containing the beam. The pole tips of the spectrometer are oriented to accept electrons emitted from the target at 125° with respect to the incident beam. Part (B) is a side view of one pole tip.

rare-earth target ($A=152$) were chosen to permit resolution in a solid state *n-p* junction detector of the oxygen ions elastically scattered from the thin target layer from those scattered from the backing. Also, backings of low atomic number were desirable to minimize the stopping electron background which increases[6] with the square of the atomic number Z. Furthermore, excited levels in low-Z materials are not expected to decay by intense emission of conversion electrons.[7,8] However, target supports of somewhat higher atomic number were required at higher bombarding energies to minimize backgrounds from nuclear reactions in the backings. Because of high background, carbon supports proved useless for bombarding energies in excess of the Coulomb barrier near 20 MeV. Aluminum was useful to 29 MeV; titanium, to 35 MeV. Copper and nickel were used for the highest bombarding energies even though they exhibited a number of conversion lines.

The targets were positioned in the vacuum chamber of a beta-ray spectrometer[9] shown schematically in Fig. 1. The oxygen beam from the tandem was focused onto the target at the object position of the spectrometer. Spectrometer alignment was checked with a tin oxide powder "target." In a minute even a weak beam produced a sharply defined brown spot on the white oxide powder. The techniques used in obtaining conversion line spectra with this spectrometer have been discussed previously.[9] For the data reported here the spectrometer was oriented to accept internal conversion electrons emitted from the target at a mean laboratory angle of 125° with respect to the incident beam. At this angle any angular distribution effects are minimized because the coefficient of the second order Legendre polynomial is zero. For the $2^+ \rightarrow 0^+$ transitions angular distribution effects are negligible.[2,9] Angular distribution effects for the $4^+ \rightarrow 2^+$ transitions are expected to be negligible also.[10] An experimental momentum resolution $\Delta P/P$ of about 2% was employed. For monitoring the incident beam, both current integration and an *n-p* junction detector were used. The high-energy resolution obtained with the detector, approximately 3% for thin targets, easily permitted discrimination between oxygen ions elastically scattered from the backing and from the target layer. A method of calculating the energy loss in the thicker targets from the width of the peak in the pulse-height spectrum of the elastically scattered ions was employed to check the energy loss calculated from stopping powers and target thicknesses obtained from the Rutherford scattering law, the integrated target current, and the elastic scattering rate. The operation of the equipment was initially checked by measuring cross sections for excitation of the first excited states in Sm^{154} and Dy^{162} with 3-MeV protons. The $B(E2)$ values obtained from

TABLE I. Oxygen ion target currents.

Ionic charge	Target current (μA) Typical	Maximum	Energy range (MeV)
3	0.4	0.8	8–13
4	1.2	2.2	12–29
5	0.5	0.8	18–39
6	0.07	0.12	39–50
7	···	<0.001	>35

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[6] T. Huus, J. H. Bjerregaard, and B. Elbek, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **30**, No. 17 (1956).

[7] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

[8] L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Reports, 1956 and 1958 [translation: Reports 57 ICC K1 and 58 ICC L1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)], Parts I and II.

[9] E. M. Bernstein and R. Graetzer, Phys. Rev. **119**, 1321 (1960).

[10] K. Alder and A. Winther (private communication).

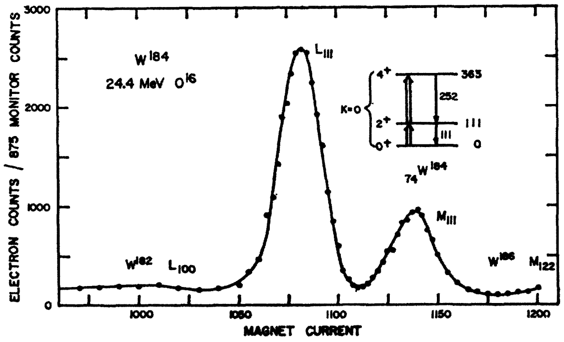

FIG. 2. Internal conversion electron spectrum observed following Coulomb excitation with 24.4-MeV oxygen ions. The target (95% W^{184}) was approximately 20 $\mu g/cm^2$ thick to the beam and was on an aluminum backing.

the cross sections were in satisfactory agreement with published $B(E2)$ values.[11] (Also see Table IV below.)

Measurements were made of the internal conversion electrons emitted following oxygen-ion induced Coulomb excitation of the first excited states in ${}_{62}Sm^{152}$, ${}_{62}Sm^{154}$, ${}_{64}Gd^{160}$, ${}_{66}Dy^{162}$, ${}_{68}Er^{170}$, ${}_{70}Yb^{176}$, ${}_{72}Hf^{180}$, ${}_{74}W^{182}$, and ${}_{74}W^{184}$. In all cases except Sm^{152} the L-shell conversion electrons were detected because of the low kinetic energy (~30 keV) of the K-shell electrons. Also for these transitions from the first excited states the probability of conversion is larger in the L shell than in the K shell. The three L subshells were not resolved. A spectrum of electron counts vs magnet current for W^{184} is shown in Fig. 2. The K-shell conversion electrons were detected for Sm^{152} because their kinetic energy is 75 keV, and in this case the K/L ratio is larger than one. For W^{182} a thin target prepared from natural tungsten oxide was used. The K-shell electrons emitted in the nuclear de-excitations from the second to the first excited states were measured in natural erbium and all the isotopes listed above with the exception of W^{182}. An electron spectrum of the double excitation in Sm^{154} is shown in Fig. 3.

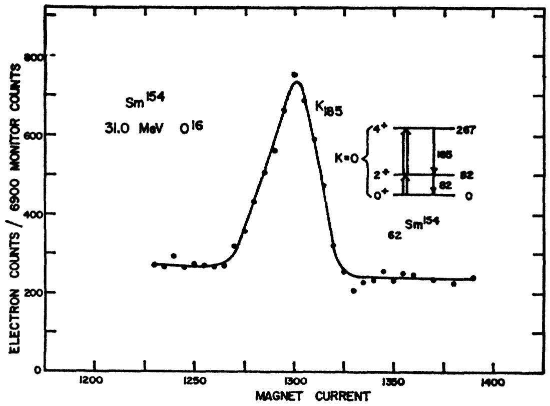

FIG. 3. Internal conversion electron spectrum observed following double Coulomb excitation with 31.0-MeV oxygen ions. The target (99% Sm^{154}) was approximately 360 $\mu g/cm^2$ thick to the beam and was on a copper backing.

Cross-section measurements in the bombarding energy range 14–50 MeV were usually made two or three times at each energy. Effects of target deterioration were minimized by inserting targets into the spectrometer so that a different region of the target was exposed to the beam for each spectrum. Cross sections were consistently reproduced at given energies with different oxygen ionic charge states, with different target support materials, and with targets of quite different thicknesses. Conversion electron yields from the thick target backings were also measured to permit correction of experimental data.

III. EXPERIMENTAL RESULTS

Energies of 4^+ levels listed in[12–14] Table II were calculated from the momenta of the observed conversion lines, the orbital binding energies of the electrons,[15] and the energies of the 2^+ levels. For each conversion line the momentum was computed from the observed dial position of the magnet current control circuit at the peak of the conversion line. The standard deviation of dial positions corresponding to the peak of a conversion line was about 0.3%. The control circuit dial was calibrated by locating the conversion lines of previously known nuclear transitions. Because of uncertainties in the calibration and in the momenta of the conversion electrons, the error in the energies of 4^+ levels is ± 2 keV.

Cross sections for Coulomb excitation were calculated from the "areas" under conversion electron peaks and the simultaneously detected yields of elastically scattered oxygen ions. The elastic scattering was assumed to be pure Rutherford scattering because the Coulomb barrier penetrability factor $1/(F_0^2+G_0^2)$ was only 3×10^{-12} for the 50-MeV (~3 MeV/nucleon) ions. De-excitation of the nuclear levels by gamma emission was taken into account by using the theoretical internal conversion coefficients.[7,8] The theoretical K and L shell conversion coefficients employed in this experiment are listed in Table III. The intensity of internal conversion in outer electron orbits was taken as 0.3 α_L although the observed ratios of $\alpha_{M+N+\dots}/\alpha_L$ were 0.28 for ${}_{66}Dy^{162}$ and 0.26 for ${}_{62}Sm^{154}$ for the $2^+ \rightarrow 0^+$ transitions. These differences are negligible in determining cross sections in this experiment.

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[11] B. Elbek, M. C. Olesen, and O. Skilbreid, Nucl. Phys. **19**, 523 (1960).

[12] E. L. Chupp, J. W. M. DuMond, F. J. Gordon, R. C. Jopson, and H. Mark, Phys. Rev. 112, 518 (1958).

[13] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.).

[14] B. M. Adams, D. Eccleshall, and M. J. L. Yates, in *Reactions Between Complex Nuclei*, edited by A. Zucker, F. T. Howard, and E. C. Halbert (John Wiley & Sons, Inc., New York, 1960), p. 95.

[15] R. D. Hill, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Appendix VI.

TABLE II. Energies of observed electron transitions.

Isotope	Observed electron transitions[a]	Other experiments $E(2^+)$ (keV)	Other experiments E_{21} (keV)	This experiment E_{21} (keV)	This experiment $E(4^+)$[h] (keV)	Rotation-vibration parameter[i] (keV)
$_{62}Sm^{152}$	K_{10},[b] K_{21}[b]	121.85[d]	244.7[e]	244.3	366.2	0.143
$_{62}Sm^{154}$	L_{10},[b] K_{21}	81.99	185[f]	185.1	267.1	0.022
$_{64}Gd^{160}$	L_{10},[b] K_{21}	75.26	...	173.3	248.6	0.008
$_{66}Dy^{162}$	L_{10}, K_{21}	80.65	185	185.3	266.0	0.010
$_{68}Er^{170}$	L_{10}, K_{21}	79.31	...	182.2[g]	261.5[g]	0.010
Er (natural)	K_{21}[c]	...	184	184.6	...	...
$_{70}Yb^{176}$	L_{10}, K_{21}	82.13	...	189.8	271.9	0.007
$_{72}Hf^{180}$	L_{10}, K_{21}[b]	93.29	215.75	216.1	309.4	0.006
$_{74}W^{182}$	L_{10}	100.07	229	...	...	...
$_{74}W^{184}$	L_{10},[b] K_{21}	111.13	253	252.3	363.4	0.025

[a] The K and L identify the shells from which the atomic electrons were ejected; the subscript 10 identifies the nuclear transition from the first excited state to the ground state; and 21 refers to the nuclear transition from the second to the first excited state.
[b] This conversion line was used for momentum calibration.
[c] Even isotopes only.
[d] Energies of all 2^+ levels are taken from reference 12.
[e] All energies in this column are taken from reference 13 except for Sm^{154}.
[f] Reference 14.
[g] This value may be too large because of the presence in the target of Er^{166} and Er^{168} which have slightly larger transition energies.
[h] The values $E(4^+)$ are the sums of entries in the third and fifth columns. The estimated errors are ±2 keV.
[i] Calculated from Eq. (2) and the energies in the third and sixth columns. The estimated errors are ±0.008 keV.

Four corrections to the cross sections were investigated. (1) Corrections were made for those conversion electrons and oxygen ions counted because of the presence of other isotopes in the isotopically enriched target materials. These corrections were less than 3% except for erbium and the sample of natural tungsten. (2) From the bombardment of copper backings conversion lines were discovered at electron kinetic energies of 60, 79, 91, 104, 114, 128, 135, 146, 158, 191 (?), and 205 (?) keV. From the nickel backings lines were observed at electron energies of 69, 73, 85, 99, 114 (?), 132, and 175 keV. Corrections were required only for the $4^+ \rightarrow 2^+$ transitions in Sm^{154} and Dy^{162} at the highest bombarding energies; but they ranged up to 17% of the observed electron yield. (3) Cross sections computed from incident projectile energies were corrected for the energy lost by the beam in penetrating the target layers. The corrections, usually less than 5%, were computed[9] from target thickness measurements and, for the thick targets, also directly from the width of the peak in the pulse-height spectrum of the elastically scattered oxygen ions. The energy losses calculated by the two methods rarely differed from their mean by more than 20%. However even a 20% uncertainty in 5% correction contributes only 1% uncertainty to the cross section. Estimates of stopping powers dE/dx for oxygen ions were derived from range-energy curves.[16] The values used for dE/dx for 50- to 20-MeV oxygen ions in the rare earths were 2.5 to 4.5 MeV/mg-cm^{-2}. (4) To obtain accurate excitation probabilities, it was necessary to correct the observed electron intensity for cascade decays from higher rotational and vibrational levels. For cascade decays from rotational levels the theoretical values of $\sigma/a^2(4^+)$ (see below) were subtracted from the experimental values of $\sigma/a^2(2^+)$ at the experimental q values (see below). Similar corrections to experimental values of $\sigma/a^2(4^+)$ for decays from the 6^+ to the 4^+ levels were also made. These corrections ranged up to 7% at the highest q values. Small corrections for the known vibrational levels[17,18] in Sm^{152} and W^{184} were also computed.

The reduced transition probabilities $B(E2; 0^+ \rightarrow 2^+)$ calculated[5] from the corrected cross sections are listed in Table IV.[11,19–22] For comparison with the predictions of Alder and Winther the cross sections σ divided by a^2 have been plotted on a semilog scale against the parameter[3] q in Figs. 4 and 5. Here a is one-half the

TABLE III. Theoretical $E2$ conversion coefficients. Conversion in higher shells was taken into account by assuming that $\alpha_{M+N+\cdots} = 0.3\alpha_L$.

Isotope	$2^+ \rightarrow 0^+$ α_K	$2^+ \rightarrow 0^+$ α_L	$4^+ \rightarrow 2^+$ α_K	$4^+ \rightarrow 2^+$ α_L
$_{62}Sm^{152}$	0.67	0.38	0.080	0.021
$_{62}Sm^{154}$	2.00	2.28	0.190	0.063
$_{64}Gd^{160}$	2.25	4.00	0.237	0.096
$_{66}Dy^{162}$	1.80	3.30	0.197	0.085
$_{68}Er^{170}$	1.67	4.25	0.215	0.102
$_{70}Yb^{176}$	1.40	4.30	0.190	0.101
$_{72}Hf^{180}$	1.03	2.75	0.135	0.068
$_{74}W^{182}$	0.85	2.35	...	...
$_{74}W^{184}$	0.69	1.43	0.089	0.042

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[16] E. L. Hubbard, University of California Radiation Laboratory Report UCRL-9053, 1960 (unpublished).

[17] R. K. Sheline, H. L. Nielsen, and A. Sperduto, Nucl. Phys. **16**, 518 (1960).

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[21] G. Goldring and Z. Vager, Nucl. Phys. **26**, 250 (1961).

[22] E. Bodenstedt, E. Matthias, H. J. Körner, E. Gerdau, F. Frisius, and D. Hovestadt, Nucl. Phys. **15**, 239 (1960).

TABLE IV. Comparison of experimental $B(E2;\,0^+\rightarrow 2^+)$ values in units of $e^2\times 10^{-48}$ cm^4.

Isotope	$B(E2)$ inelastic scattering	$B(E2)$ lifetime	Weighted[f] $B(E2)$	$B(E2)$ conversion electrons	Q_0[i] (10^{-24} cm^2)
Sm^{152}	3.53±0.10[a] 3.40±0.15[b]	3.4±0.1[d]	3.46±0.08	3.67±0.25[g]	5.90
Sm^{154}	4.61±0.20[b]	4.7±0.4	4.63±0.18	4.38±0.30[g] 4.53±0.35[h]	6.82
Gd^{160}	5.80±0.25[b]	5.5±0.3	5.68±0.19	5.43±0.40	7.56
Dy^{162}	5.11±0.15[b]	5.5±0.3	5.19±0.13	4.68±0.35 4.80±0.35[h]	7.22
Er^{170}	5.44±0.15[b]		5.44±0.15	6.13±0.45	7.40
Natural Er					7.52[j]
Yb^{176}	5.78±0.20[b]	4.9±0.3	5.51±0.17	5.28±0.40	7.44
Hf^{180}	4.35±0.20[c]		4.35±0.20	4.93±0.35	6.61
W^{182}	4.00±0.20[c]	4.3±0.3	4.09±0.17	4.58±0.40	6.41
W^{184}	3.62±0.20[c]	3.6±0.2 3.66±0.23[e]	3.62±0.12	4.18±0.30	6.03

[a] Reference 19.
[b] Reference 11.
[c] Reference 20.
[d] Unless otherwise noted, the $B(E2)$ values in this column are taken from reference 21.
[e] Reference 22.
[f] The experimental values in the second and third columns were weighted inversely proportional to the squares of the standard errors quoted.
[g] The $B(E2)$ values calculated from data obtained at 45 MeV are not included. See the discussion in text.
[h] Calculated from data obtained with 3-MeV protons.
[i] With the exception of natural erbium the quadrupole moments were calculated from equation 1 of the text and the weighted $B(E2)$ values listed in this table.
[j] The quadrupole moments for the even-A erbium isotopes were taken from reference 11 and weighted in proportion to the relative natural abundance.

distance of closest approach in a head-on collision and q, which depends on the quadrupole moment Q_0 of the deformed nucleus,[5] is proportional to the $\frac{3}{2}$ power of the bombarding energy. The lowest point in Fig. 4 and the two highest points in Fig. 5 are each based on single measurements. The solid lines in Figs. 4 and 5 are based on a numerical calculation[23] of multiple Coulomb excitation cross sections.

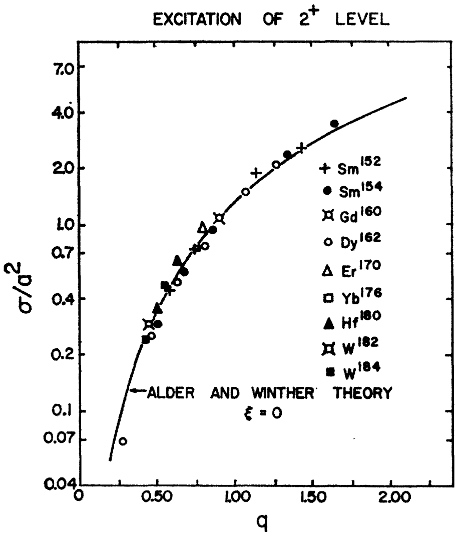

FIG. 4. Excitation of the 2^+ rotational level of the ground-state band compared with the Alder-Winther theory. The estimated uncertainty is about 7% in values of σ/a^2 and about 3% in values of q. The Dy^{162} point plotted at $q=0.276$ is based on a single measurement.

Two major sources of error in the electron yields limit the precision of $B(E2;0^+\rightarrow 2^+)$ and $\sigma/a^2(2^+)$ values to about 7%. There is an error of about 5% in subtraction of background beneath conversion peaks and an error of nearly 5% in the spectrometer transmission factor. For the weak $4^+\rightarrow 2^+$ transition the uncertainty in background subtraction is about 9% and the estimated error in values of $\sigma/a^2(4^+)$ is about 13%. An uncertainty of 3% in q arises primarily from the error in Q_0. Because Q_0 is related[5] to the $B(E2)$ value for even-even nuclei by

$$B(E2;0^+\rightarrow 2^+)=(5/16\pi)e^2Q_0^2, \qquad (1)$$

the error in q is based on the errors in $B(E2;0^+\rightarrow 2^+)$ values.

IV. DISCUSSION

For a deformed even-even nucleus Bohr and Mottelson[5] predicted a spectrum of rotational energy levels E_I given by

$$E_I=\hbar^2I(I+1)/2\mathscr{I}-bI^2(I+1)^2,\quad I=0,2,4,6\cdots, \qquad (2)$$

where I is the nuclear spin quantum number, $\mathscr{I}$ is the moment of inertia, and b is a measure of the rotation-vibration interaction. Values of this rotation-vibration parameter computed from $E(4^+)$ and $E(2^+)$ are listed in Table II. For all cases the uncertainty in b is ±0.008 keV. The values of $E(4^+)$ and b reported here are consistent with the values observed in other deformed even-even nuclei.[24]

The $B(E2)$ values obtained in this experiment (Table IV) are in agreement with other determinations of $B(E2)$ values. The data on excitation of the 2^+ levels

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[23] R. Graetzer, R. Hooverman, and E. M. Bernstein, Nucl. Phys. (to be published).

[24] R. K. Sheline, Rev. Mod. Phys. 32, 1 (1960).

(Fig. 4) support the theory of Alder and Winther. At the lower bombarding energies the perturbation calculations and the newer theory are indistinguishable.[3] At the higher bombarding energies the Alder-Winther cross sections are lower than the values predicted by first order perturbation theory. Because the perturbation approximation is not expected to be correct at 45 MeV, the $B(E2)$ values calculated from cross sections measured at this energy were not included in the $B(E2)$ values presented in Table IV. For Sm^{154} the calculated $B(E2)$ value at 45 MeV is 5% lower than the quoted average obtained from measurements at lower bombarding energies. At 45 MeV the $B(E2)$ value for Sm^{152} is 15% lower. These deviations tend to support Alder and Winther's theory.

The data shown in Fig. 5 provide qualitative support for their theory of double Coulomb excitation. For eight different isotopes in the range $Z=62$ to $Z=74$ the data exhibit the predicted variation with q over more than two orders of magnitude of $\sigma/a^2(4^+)$. Data from a target of natural erbium are similar to the data obtained from Er^{170}. This agreement suggests that Er^{166} and Er^{168}, which have energy level schemes and quadrupole moments similar to those of Er^{170}, also support the theory. Of special interest is the fact that the Sm^{152} data also agree with the theory[15] even though Sm^{152} with only 90 neutrons does not have the typical rotational spectrum. The systematic deviation from theory of all data points for low q values probably results from the use of $\xi=0$ in the theoretical calculations.[10] The values of ξ in this experiment ranged from 0.06 at the highest bombarding energies (highest q values) to 0.3 for the lowest.

Particularly significant are the extensive data on excitation of the 4^+ levels in Sm^{154} and Dy^{162}. These isotopes have quite similar energy levels and quadrupole moments. However, for intermediate q values a smooth curve drawn through the values of σ/a^2 for Dy^{162} lies 20–30% above a similar curve sketched through the Sm^{154} data. The estimated experimental error in this ratio is about 16%. This systematic difference in cross section as a function of q may result from (a) inaccuracies in theoretical conversion coefficients, (b) deviations from the Bohr-Mottelson collective model on which the Alder-Winther theory is based, (c) errors in cross sections resulting from cascade decays from unknown vibrational levels, and/or (d) errors in q resulting from errors in the quadrupole moments Q_0.

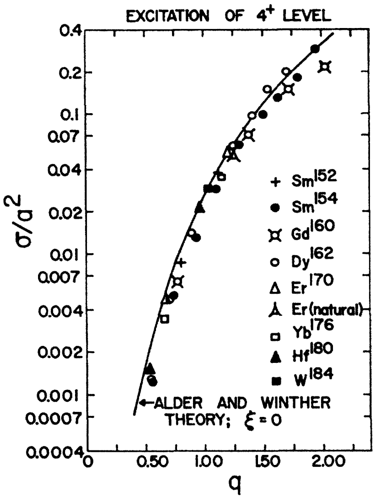

FIG. 5. Experimental results of double $E2$ Coulomb excitation compared with the Alder-Winther theory. The estimated uncertainty is about 13% in values of σ/a^2 and about 3% in q. The Sm^{154} and Gd^{160} points at $q=1.95$ and $q=2.03$, respectively, are each based on single determinations of the cross sections.

Although experimental $E2$ conversion coefficients may deviate systematically from theoretical coefficients,[25] the experimental errors are large. Also direct measurements of lifetimes of 4^+ rotational levels[26] in Dy^{162}, Er^{168}, and Hf^{180} provide evidence supporting the Bohr-Mottelson model. Finally, the existing information about vibrational states is inadequate to predict energy levels and transition probabilities to determine accurately the influence of cascade decays on the measurements of $\sigma/a^2(4^+)$.

The results of the present experiment can be summarized briefly. (1) $B(E2)$ values for excitation of the first rotational 2^+ levels of nine rare earth isotopes are in reasonable agreement with results obtained in other laboratories. (2) Energies of 4^+ rotational levels have been determined with errors of ±2 keV for eight rare-earth isotopes. (3) Experimental data obtained with oxygen ions generally support the Alder-Winther theory. The cross sections for excitation of the 2^+ levels of Sm^{152} and Sm^{154} at 45 MeV are in satisfactory agreement with the Alder-Winther theory and are slightly smaller than predicted by first-order perturbation theory. A discrepancy between theory and experiment for low q values is probably due to the approximation $\xi=0$ in the theoretical calculations. An apparent systematic difference in the values of $\sigma/a^2(4^+)$ for Dy^{162} and Sm^{154} has not been explained.

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